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LETTER TO THE EDITOR

Diffusion of saltating particles in unidirectional water flow over a rough granular bed

Vladimir Nikora[1], **John Heald**[2], **Derek Goring**[1] **and Ian McEwan**[2]

[1] National Institute of Water and Atmospheric Research (NIWA), PO Box 8602, Christchurch, New Zealand

[2] Department of Engineering, University of Aberdeen, Aberdeen AB24 3UE, UK

E-mail: v.nikora@niwa.cri.nz

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Abstract

We show that the motion of saltating particles on a flat rough bed in unidirectional water flow is diffusive and comprises three ranges (local, intermediate, and global) of spatial and temporal scales with different scaling behaviour and diffusion properties. Our computer simulations suggest that the ratio of the travelling particle diameter to the prevailing diameter of static bed particles (or the height of bed roughness) is one of the key parameters controlling particle diffusion.

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1. Introduction

For the last 10–15 years the dynamics of granular materials has opened new frontiers for understanding complex nonlinear systems and thus has attracted much attention from physicists and engineers [1]. Indeed, sand avalanches and clouds of flowing grains demonstrate exciting nontrivial behaviour and serve as metaphors for other complex systems like turbulence or plasma transport [1, 2]. Even the simplest case of individual particle dynamics (i.e., without involvement of collective behaviour of many grains) shows rich and nontrivial physics. For instance, gravity-driven motion of a particle on an inclined rough plane shows ordinary diffusion [3] while downhill motion of a particle within continuously moving avalanches on the sandpile slope demonstrates super-diffusion [2]. In this letter we present an additional example of complex behaviour of individual grains which has not been reported yet, i.e., diffusion of saltating grains at small and intermediate scales in unidirectional water flow over a flat rough granular bed. One may note, for curiosity, that the term 'saltation' originates from the Latin 'saltari' meaning to dance.

Depending on the near-bed flow velocity, a solid particle may either rest on the bed or be in one of the following modes of motion: (1) sliding/rolling mode; (2) saltation (hopping/bouncing) mode; or (3) suspension mode [4–6]. The change from one mode to another occurs spasmodically as a result of increase in flow velocity (or in the bed shear stress). In other words, there are several velocity thresholds which set ranges for these modes of grain motion. Furthermore, increase in flow velocity (or in bed shear stress) may also lead to instability of the flat granular bed and to development of ripples and dunes. In general, the following transport stages of sediment movement which replace each other with increase in flow velocity may be distinguished: (1) flat bed without moving particles (very low, subcritical, flow velocity); (2) flat bed with moving particles; (3) ripples; (4) dunes; (5) flat bed with intense 'massive' movement of bed particles; and (6) antidunes [5, 6]. It is believed that the sliding/rolling mode and saltation (hopping/bouncing) mode of particle motion may occur for transport stages 2–4 [5, 6]. The transport stages 3 and 4 (ripples and dunes) have already been extensively studied in physics literature, at least for the case of shear water flows [7], oscillatory water flows [8, 9], and air flows [10, 11]. In this letter we consider the least studied mode of particle saltation (hopping/bouncing) for stage 2, i.e., the motion of saltating particles on the flat rough bed in unidirectional water flow.

Hans Albert Einstein, the older son of the great Albert Einstein, was probably the first who approached the sediment transport problem from a statistical point of view and developed a stochastic theory for sliding/rolling and saltation modes of particle motion which define the so-called bed-load [12]. This theory is regarded as one of the most significant advances in sediment transport research during the last century. One of the key assumptions of his theory was that 'bed-load movement is to be considered as the motion of bed particles in quick steps with comparatively long intermediate periods of rest' [12, p 563]. Because Einstein's prime objective was to derive a relationship for the streamwise sediment flux he analysed only longitudinal (along the flow) components of particle displacements, neglecting any transverse movements. Thus, he considered particle motion to consist of 'instantaneous' steps, which we term here *intermediate trajectories*, separated by relatively long rest periods. We define the particle trajectory consisting of many intermediate trajectories as the *global trajectory*. The behaviour of particle motion at the scale of this global trajectory was the main point in Einstein's considerations. The details of 'instantaneous' particle motion from one rest position to another (i.e., at the scale of the intermediate trajectory) were not essential for his derivations. In reality, however, particle motion between rest positions can occur in either sliding/rolling mode or saltation mode, or a mixture of them. In the following considerations we assume, for simplicity, that a particle moves from one rest position to another in saltation mode when forces due to fluid drag and particle weight dominate motion. Most previous studies of bed particle saltation concentrated on a description of the trajectory between two successive collisions with the bed, i.e., the ballistic particle trajectory [4–6]. We define this small-scale particle trajectory as the *local trajectory*. Note that the intermediate trajectory consists of many local trajectories just as the global trajectory consists of many intermediate trajectories. In other words, the intermediate particle trajectory may include tens or hundreds of collisions with the bed. Thus, we suggest in this letter that particle motion occurs within at least three ranges of scales, which we conceptually define as local, intermediate, and global.

In physical applications bed particle motion has been studied, to a certain degree, at the local trajectory scale and also at the global trajectory scale [4–6, 12]. However, the dynamics of particle motion between rest periods, i.e., at the scale of the intermediate trajectory, has been neglected for the sake of simplification and largely overlooked in considerations. In this letter we attempt to fill this gap. In particular, we consider the projection of the intermediate trajectory, which is three dimensional by nature, on the plane parallel to the bed, with x-axes along the flow and y-axes across the flow. We believe that this projection provides intriguing information about the particle dynamics in unidirectional flow missed in previous studies.

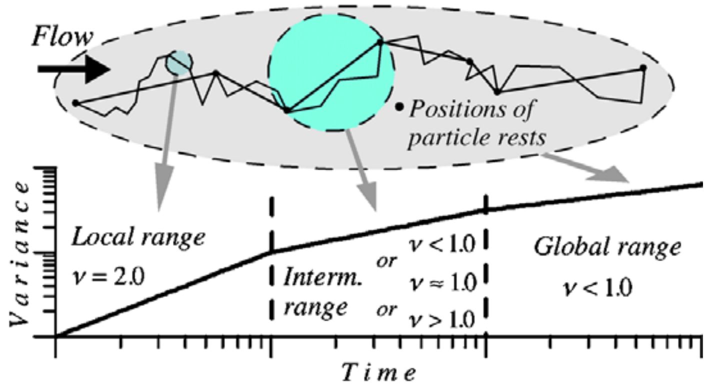

Figure 1. A plan view of a bed particle trajectory consisting of three distinct ranges of scales (local, intermediate, and global) with different scaling behaviours. Note that for simplicity the local trajectories (small circle) are shown by straight segments. However, in general, the horizontal projections of the local trajectories are more likely to be smoothly curved.

(This figure is in colour only in the electronic version)

2. Concept

Figure 1 shows a schematic global trajectory of a bed particle which illustrates the scale considerations presented above. Differences in the physical processes which govern the local, intermediate, and global trajectories give rise to complex scaling behaviour which, we believe, is treated holistically here for the first time. In general, in figure 1 we suggest that the motion of saltating particles in unidirectional flow is diffusive and comprises three ranges (local, intermediate, and global) of spatial and temporal scales with different scaling behaviour and diffusion properties. Indeed, the local trajectories are smooth (nonfractal) and, thus, one would expect the local range diffusion to be ballistic, i.e., the variances of particle coordinates $[X, Y]$ increase in time as $\langle X'^2\rangle \propto \langle Y'^2\rangle \propto t^{\nu=2}$ where $X' = X - \langle X\rangle$, $Y' = Y - \langle Y\rangle$, and $\langle X\rangle$, $\langle Y\rangle$ are mean values. For the intermediate range of scales the diffusion may be either slow diffusion ($\langle X'^2\rangle \propto \langle Y'^2\rangle \propto t^{\nu<1}$), normal diffusion ($\langle X'^2\rangle \propto \langle Y'^2\rangle \propto t^{\nu=1}$), or super-diffusion ($\langle X'^2\rangle \propto \langle Y'^2\rangle \propto t^{\nu>1}$), depending on what factors dominate. For instance, the bed topography and near-bed turbulence may have opposite effects on bed particle diffusion. A 'fractal' bed may try to slow down diffusion processes ($\nu < 1$) [13] while turbulence may enhance them ($\nu > 1$) [14], or they can mutually cancel their effects ($\nu = 1$). The particle behaviour in the global range of scales is most probably sub-diffusive ($\nu < 1$), as a result of potentially infinite rest periods [2, 14]. Figure 1 summarizes our concept. Results of this study relate, mainly, to the local and intermediate ranges of scales. Specifically, using computer simulations we study the effects of the mean flow velocities, near-bed turbulence, and relative particle diameter on particle diffusive properties and particle trajectories. We define the ratio of the travelling particle diameter to the prevailing diameter of static bed particles as the relative particle diameter d_r. The travelling particle may change direction dramatically only if it collides with a similar or larger static bed particle. Otherwise the particle path may not be affected appreciably by collisions with the bed. This consideration suggests that the boundary between the local and intermediate ranges of scales in figure 1 should depend on characteristic time and length scales between these significant collisions rather than on the mean time and distances between all collisions. We show below that this is indeed the case and therefore the relative particle size is important.

3. Computer simulations

In our simulations we used the discrete particle model of Jefcoate and McEwan [15] developed for simulations of particle behaviour in fluid flows. The model is based on the molecular dynamics approach first introduced for granular material simulations by Cundall and Strack [16]. The essential features of the model are: (1) Newton's equations of particle motion; (2) the main acting forces are drag, lift, and buoyancy; (3) if a grain is found to rebound repeatedly off three settled grains and it is in static equilibrium when brought into contact with those three grains, then this contacting position is adopted as the grain's settling position. Thus, trajectories and velocities of discrete spherical particles are obtained by integrating Newton's equations of particle motion at small time steps (<0.01 s in this study). An individual grain may take two roles during a simulation: moving freely according to the above rules or lying at rest.

For the purposes of this study we modified Jefcoate and McEwan's [15] model by introducing a more realistic representation of near-bed turbulence. The fluid flow in the modified model is represented as two dimensional, i.e., time and spatially averaged flow properties depend on the vertical coordinate only. Two dimensionality also means that the time and spatially averaged vertical and transverse velocities are zero while the vertical distribution of the mean longitudinal velocity is linear within the roughness sublayer (below tops of surface grains), and logarithmic above the roughness sublayer [17–19]. To simulate turbulent velocity fluctuations we used a model of velocity spectra [18, 20], which consists of four ranges: (1) the range of the largest eddies, where turbulence energy production occurs; (2) the range of intermediate eddies, where energy production and cascade energy transfer coexist [20]; (3) Kolmogorov's inertial sub-range of relatively small eddies; and (4) the viscous range. The model's simulated space was broken up into many longitudinal strips and velocity time series were produced for each strip, using the Monte Carlo approach. The transverse correlations between velocity time series were simulated by correlating spectral phases of velocity signals across the flow. This procedure allowed us to produce a near-bed velocity field with the desired transverse correlation length scale. The main parameters of the model which control both the mean velocities and turbulence are the shear velocity (or bed shear stress), the thickness of the roughness sublayer, the flow depth, and the transverse correlation length. The flow properties (i.e., mean velocities, turbulence intensities, and velocity spectra) in our simulations were nearly identical to those observed in real mobile-bed open-channel flows [18].

The simulations comprised three stages. In stage 1, an initial bed 1000 mm by 10 000 mm was constructed of 1.5 million grains, released with zero velocity one at a time from a random horizontal position. A log-normal grain size distribution of between 2 and 8 mm with a d_{50} of 4 mm was used. The fluid velocity was set to zero to simulate particle deposition in still water. This bed was then used as a surface upon which a further 1.5 million grains could be settled under unidirectional flow conditions in stage 2. This latter stage was repeated for various simulation scenarios to produce twelve simulated granular beds, each of three million grains. In our simulation scenarios we varied the shear velocity (or the bed shear stress), the transverse correlation length for flow velocities, and the number of velocity components. Thus, there were scenarios where vertical velocity or transverse velocity or both were switched off, keeping the longitudinal velocity unchanged. The statistical properties of 'water-worked' beds appeared to be similar (the bed elevation standard deviation σ_z: 2.3 $\pm$ 0.05 mm; skewness: -0.67 ± 0.06; kurtosis: 1.25 ± 0.19) but distinct from those for the deposited bed in stage 1 (σ_z: 2.08 $\pm$ 0.05 mm; skewness: 0.07 ± 0.06; kurtosis: -0.37 ± 0.19). Stage 3 involved consecutively releasing 500 individual grains of each size from 1 to 10 mm at the upstream edge of each of the beds produced in stage 2 and recording their trajectories as they saltated

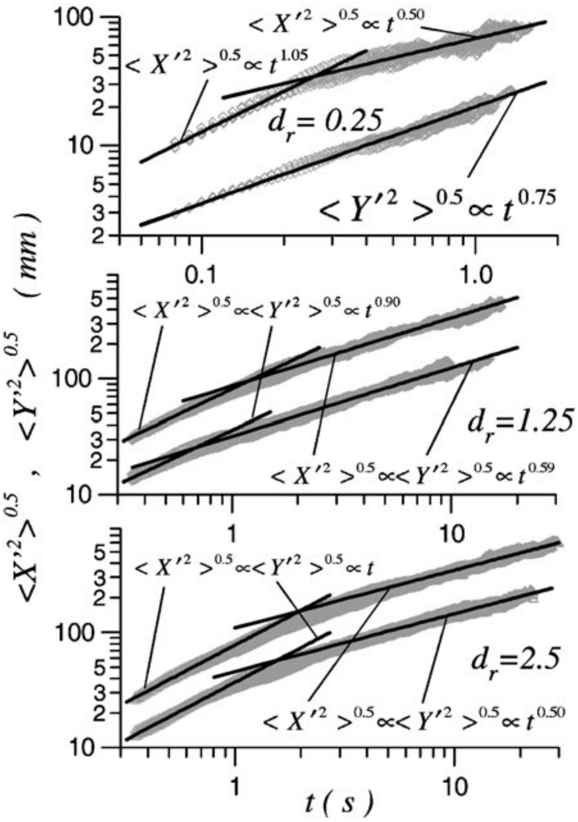

Figure 2. Diffusion plots, $\langle X'^2 \rangle^{0.5}$, $\langle Y'^2 \rangle^{0.5} = f(t)$, for three representative particle sizes (the shear velocity is 99 mm s^{-1}).

downstream and either deposited or left the simulation area. The model parameters of each scenario in stage 2 were preserved into stage 3, so their effects upon the grains' trajectories, if any, could be observed. The grains were released individually and allowed to settle or exit before the next grain was released. Thus, we could study the particle trajectories formed by particle–bed collisions in pure form, i.e., without complications due to collisions between travelling particles. $d_{50} = 4$ mm was used as the prevailing diameter of the static bed particles in calculations of the relative particle diameters d_r.

4. Particle diffusion and trajectories

Using the model described above, we explored various properties of particle motion. The most interesting results relate to particle diffusion and trajectories. Analyses of moments $\langle X'^q \rangle$, $\langle Y'^q \rangle = f_q(t)$ for $q = 2$, 3, and 4 revealed that the bed particle motion within the local and intermediate ranges of scales (figure 1) is indeed diffusive, i.e., the function $f_q(t)$ may be approximated as $f_q(t) \propto t^{\nu_q}$ with ν_q dependent mainly on the relative particle size and the diffusion time. The linearity of the relationship $\nu_q = q\gamma(q)$ for our simulations suggests that the observed diffusion does not belong to the class of strong anomalous diffusion [21] and, thus, may be characterized by a single exponent $\gamma(q) \equiv \gamma$. Figure 2 shows plots of $\langle X'^2 \rangle^{0.5}$, $\langle Y'^2 \rangle^{0.5} = f(t)$ for three representative particle diameters, 1, 5, and 10 mm, which

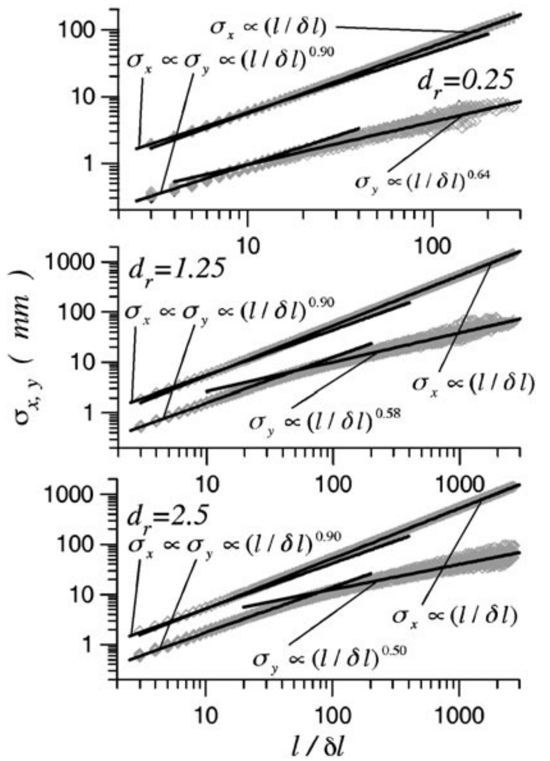

Figure 3. Matsushita–Ouchi plots, σ_x, $\sigma_y = f(t)$, for three representative particle sizes. The sampling interval δl along a trajectory is 2 mm (the shear velocity is 99 mm s^{-1}).

correspond to $d_r = 0.25$, 1.25, and 2.5, respectively. We found no evidence of turbulence effects on the diffusion plots $\langle X'^2\rangle^{0.5}$, $\langle Y'^2\rangle^{0.5} = f(t)$ as data for different turbulence properties were mixed and collapsed within fairly narrow bands. In contrast, the diffusion time and the relative diameter d_r of the travelling particles seem to be the main factors influencing diffusion properties. The local range of scales with the ballistic diffusion ($\nu = 2$ or $\gamma = 1$), postulated in figure 1, appears in all plots (independently of d_r) except the plot for $\langle Y'^2\rangle^{0.5} = f(t)$ with $d_r = 0.25$. This deviation from the general rule is, most probably, due to resolution problems. As for the intermediate range of scales, figure 2 suggests that at least three different diffusion regimes may exist: (1) normal diffusion ($\nu = 1$ or $\gamma = 0.5$) in the x-direction and anomalous (super-) diffusion ($\nu > 1$ or $\gamma > 0.5$) in the y-direction for $d_r \ll 1$; (2) anomalous (super-) diffusion ($\nu > 1$ or $\gamma > 0.5$) in both directions for $d_r \approx 1$; and (3) normal diffusion ($\nu = 1$ or $\gamma = 0.5$) in both directions for $d_r \gg 1$. The boundary between the local range and the intermediate range, t_b, shifts towards larger diffusion times with an increase in d_r as, approximately, $t_b \propto d_r$ (figure 2). Also, in all cases longitudinal diffusion is stronger than transverse diffusion. These results find support in scaling analysis of the particle trajectories. We used a method suggested by Matsushita and Ouchi [22] for the analysis of self-affine curves. The method is based on the consideration of the relationships $\sigma_x \propto l^{\zeta_x}$ and $\sigma_y \propto l^{\zeta_y}$ where σ_x and σ_y are mean standard deviations of trajectory coordinates within the distance l measured along the trajectory, ζ_x and ζ_y are scaling exponents. For a self-similar trajectory, $\zeta_x = \zeta_y = (1/D_f)$ where D_f is the fractal (self-similarity) dimension of the trajectory. In the case of self-affinity we have $\zeta_x \neq \zeta_y$ and so instead of a single exponent D_f we should use two exponents ζ_x and ζ_y or, alternatively, the lacunarity dimension $D_G = 2/(\zeta_x + \zeta_y)$ (which characterizes the degree of lacunarity or noncompactness of a particle trajectory) and the Hurst exponent $H = \zeta_y/\zeta_x$ characterizing the trajectory's self-affinity. Figure 3 shows Matsushita–Ouchi plots which suggest self-similarity at small scales (the local range) and self-affinity at large scales (the intermediate range). As in the case of $\langle X'^2\rangle^{0.5}, \langle Y'^2\rangle^{0.5} = f(t)$, the plots $\sigma_x, \sigma_y = f(l)$ and scaling exponents ζ_x and ζ_y depend on the scale of consideration and the relative particle diameter d_r, but not on specific properties of near-bed turbulence. The relatively large values of l_b, which is a length scale separating self-similarity and self-affinity regimes in figure 3, and their dependence on d_r support the idea that the position of this boundary depends on the characteristic length between significant collisions with the bed rather than on the mean distance between all collisions. In other words, travelling particles with different d_r 'feel' the granular bed differently and, therefore, the relationship $l_b, t_b = f(d_r)$ may be viewed not only as a diffusion characteristic but also as a measure of bed topography.

5. Conclusions

In conclusion, in this letter we presented a new concept for diffusion of saltating particles in unidirectional water flow over a rough granular bed which is supported by computer simulations. We believe that a similar concept should also be valid for the sliding/rolling mode of particle motion and we have been working on this problem.

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